

# Fairwashing SHAP

or

# Interventional and Observational Shapley Values

---



# Motivation

Someone walks into a bank, applied for a loan, gets rejected.

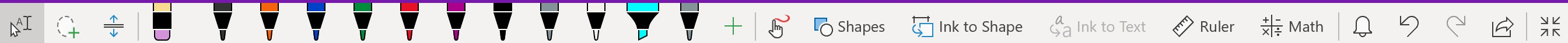
Why?

2 Kinds of explanations:

Recourse: What could they do to get the loan?

→ R\AI Developer: Why did the model say no? It is bigoted?  
Focus on this





# Fairwashing

Suppose  $f$  is very racist.

Then  $SHAP(f, \mathcal{F})$  should show that race is an important feature

But, we can make ML model  $\tilde{f}$  such that

- ① For almost all  $\mathcal{F}$   $\tilde{f}(\mathcal{F}) = f(\mathcal{F})$
- ②  $SHAP(\tilde{f}, \mathcal{F})$  shows almost no important for race

$\tilde{f}$  is a "**Fairwashed**" version of  $f$ . It looks fair, but it ain't!

- At least 2 papers: Umang's paper, [Slack et al. 2020]
- But, how is this possible? What about game theory?



# Takeaway

Q: How is fairwashing possible? What about game theory?

A: "Interventional" vs "Observational" Shapley Values



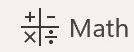
What SHAP and QII do  
Bad for proxys



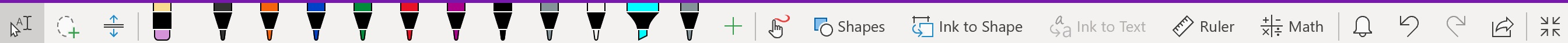
Other papers do this.  
Good for proxys.

"True to the Model or True to the Data" (on arXiv)

↳ Great jumping off point, easy read



SHAPLEY VALUES



## Some notation

- We have  $d$  features:  $\underline{x}_1, \dots, \underline{x}_n, \underline{q} \in \mathbb{R}^d$
- Let  $S \subseteq \{1, 2, 3, \dots, d\}$  be a subset of features
- Let  $\bar{S}$  complement of  $S$
- Let  $(\underline{q})_S$  = entries of  $\underline{q}$  indexed by  $S$
- Notice:  $\underline{q} = (\underline{q})_S + (\underline{q})_{\bar{S}}$
- Frankenstein Point:  $(\underline{q})_S + (\underline{x}_j)_{\bar{S}}$

$$d=5$$

$$\underline{q} = [9 \ 7 \ 5 \ 3 \ 1]$$

$$S = \{1, 2\}$$

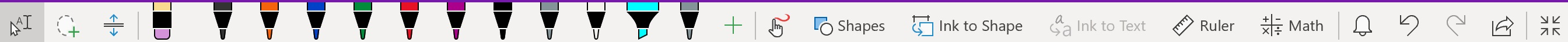
$$\bar{S} = \{3, 4, 5\}$$

$$(\underline{q})_S = [9 \ 7 \ 0 \ 0 \ 0]$$

$$(\underline{q})_{\bar{S}} = [0 \ 0 \ 5 \ 3 \ 1]$$

$$\underline{x}_1 = [2 \ 2 \ 2 \ 2 \ 2]$$

$$(\underline{q})_S + (\underline{x}_1)_{\bar{S}} = [9 \ 7 \ 2 \ 2 \ 2]$$



# Shapley Values

Not "Quantity of Interest"

Let  $v_S(\phi)$  be the **Value** of  $\phi$  with respect to  $S$ .

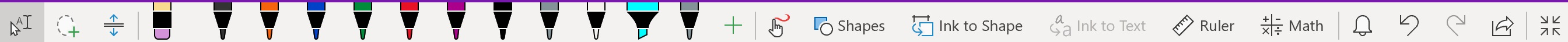
The average marginal contribution of feature  $i$ .

$$\Phi_i(\phi) := \underbrace{\sum_{\substack{S \subseteq \{1, \dots, d\} \\ i \notin S}} \frac{1}{d} \cdot \frac{1}{\binom{d-1}{|S|}}}_{\text{Average over } S} \underbrace{\left( v_{S \cup \{i\}}(\phi) - v_S(\phi) \right)}_{\text{How much } i \text{ increases value, starting from } S}$$

① **Interventional**  $v_S(\phi) = \mathbb{E}_{\underline{x} \sim \mathcal{D}} \left[ f \left( (\phi)_S + (\underline{x})_{\bar{S}} \right) \right] \approx \frac{1}{n} \sum_{j=1}^n f \left( (\phi)_S + (\underline{x}_j)_{\bar{S}} \right)$

- Used by SHAP (by default), QII
- Creates very fake-looking data points (ignores dependencies)





① **Interventional**  $\nu_s(\varphi) = \mathbb{E}_{\underline{x} \sim \mathcal{D}} [f((\varphi)_s + (\underline{x})_{\bar{s}})] \approx \frac{1}{n} \sum_{j=1}^n f((\varphi)_s + (\underline{x}_j)_{\bar{s}})$

- Used by SHAP (by default), QII
- Creates very fake-looking data points (ignores dependencies)

② **Observational**  $\nu_s(\varphi) = \mathbb{E}_{\underline{x} \sim \mathcal{D}} [f((\varphi)_s + (\underline{x})_{\bar{s}}) \mid (\underline{x})_s = (\varphi)_s]$   
 $\approx$  Average of  $(\varphi)_s + (\underline{x}_j)_{\bar{s}}$  for all  $(\underline{x}_j)_s = (\varphi)_s$  ?

- Less used, but is used
- Creates real-looking data points
- Hard to compute (few points to average)

How can we compare these 2 approaches?



## Simple Linear Model

Suppose  $f(q_f) = \langle \underline{w}, q_f \rangle + \mathcal{B}$  for some  $\underline{w} \in \mathbb{R}^d$ ,  $\mathcal{B} \in \mathbb{R}$ .

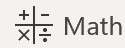
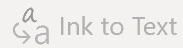
How do interventional & observational  $\Phi_i(q_f)$  look?

Lemma

Interventionally,

$$\Phi_i(q_f) = w_i (q_i - \overset{\text{average of feature } i}{\mu_i})$$

Takeaway: Completely ignores proxy variables



Takeaway: Completely ignores proxy variables

$\underline{x} = (\text{Kindness}, \text{salary}, \text{age})$  where  $\text{salary} = 1000 \cdot \text{age}$

$\underline{w}_{\text{BAD}} = [1, 0, 1000]$  is unfair: uses age a lot

$\Phi_{\text{AGE}}(\underline{q}) = 1000(q_i - \mu_i) \neq 0$ , great!

$\underline{w}_{\text{WASHED}} = [1, 1, 0]$  makes exact same predictions

$\Phi_{\text{AGE}}(\underline{q}) = 0$ , terrible!





**Lemma Proof:**  $\phi_i(\underline{q}) = w_i (q_i - \mu_i)$  interventionally

$$\mathcal{V}_s(\underline{q}) = \frac{1}{n} \sum_{i=1}^n f((\underline{q})_s + (\underline{x}_j)_s) = \frac{1}{n} \sum_{i=1}^n \langle \underline{w}, (\underline{q})_s + (\underline{x}_j)_{\{i\}} + (\underline{x}_j)_{S \setminus \{i\}} \rangle + \mathcal{R}$$

$$\mathcal{V}_{s \cup \{i\}}(\underline{q}) = \frac{1}{n} \sum_{i=1}^n f((\underline{q})_{s \cup \{i\}} + (\underline{x}_j)_{S \setminus \{i\}}) = \frac{1}{n} \sum_{i=1}^n \langle \underline{w}, (\underline{q})_s + (\underline{q})_{\{i\}} + (\underline{x}_j)_{S \setminus \{i\}} \rangle + \mathcal{R}$$

$$\mathcal{V}_{s \cup \{i\}}(\underline{q}) - \mathcal{V}_s(\underline{q}) = \frac{1}{n} \sum_{i=1}^n \langle \underline{w}, (\underline{q})_{\{i\}} - (\underline{x}_j)_{\{i\}} \rangle$$

$$= \langle \underline{w}, (\underline{q})_{\{i\}} - (\underline{\mu})_{\{i\}} \rangle$$

$$[0 \dots 0 (q_i - \mu_i) 0 \dots 0]$$

$$= w_i (q_i - \mu_i)$$

$$\phi_i(\underline{q}) = \text{Average of } \mathcal{V}_{s \cup \{i\}}(\underline{q}) - \mathcal{V}_s(\underline{q}) \text{ across all } = w_i (q_i - \mu_i) \quad \blacksquare$$



# Beyond Intervensional Value

Above proof does not work for observation values

$$\mathcal{V}_S(q) = \mathbb{E}_{\underline{x} \sim \mathcal{D}} [f((q)_S + (\underline{x})_{\bar{S}}) \mid (\underline{x})_S = (q)_S]$$

Only Frankenstein  $q$  with  $\underline{x}_j$ 's that match  $q$  on  $S$ .

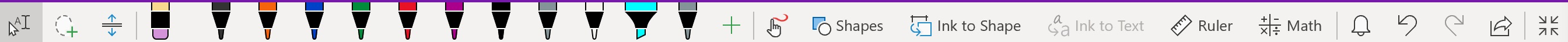
So,

$\mathcal{V}_{S \cup \{i\}}(q) - \mathcal{V}_S(q)$  depends on  $S$ , breaking the proof.

Claim

If  $f(q) = \langle w, q \rangle + b$  and  $\underline{x}_j \stackrel{iid}{\sim} \mathcal{N}(\underline{\mu}, \Sigma)$ , then we can write  $\phi_i(q)$  exactly (but it's ugly).

But it super depends on correlation in  $\Sigma$ !



## Comparison

Let

$$\underline{x}_j \sim \mathcal{N}\left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & & \\ & 1 & 0.99 \\ & 0.99 & 1 \end{bmatrix}\right)$$

Features 2 and 3  
are super correlated

$$\underline{w} = [1 \ 2 \ 3], \quad \underline{q} = [1 \ 1 \ 1]$$

Interventional

$$\Phi_1(\underline{q}) = 1$$

$$\left. \begin{aligned} \Phi_2(\underline{q}) &= 2 \\ \Phi_3(\underline{q}) &= 3 \end{aligned} \right\} \text{Treated} \\ \text{Very} \\ \text{Different}$$

Observational

$$\Phi_1(\underline{q}) = 1$$

$$\left. \begin{aligned} \Phi_2(\underline{q}) &\approx 2.5 \\ \Phi_3(\underline{q}) &\approx 2.5 \end{aligned} \right\} \text{Treated} \\ \text{the} \\ \text{Same!}$$

## Interventional

$$\phi_1(q_f) = 1$$

$$\phi_2(q_f) = 2$$

$$\phi_3(q_f) = 3$$

Treated  
Very  
Different

$$w = [1 \ 0 \ 3]$$

If  $w_2 = 0$ ,

Then  $\phi_2(q_f) = 0$

## Observational

$$\phi_1(q_f) = 1$$

$$\phi_2(q_f) \approx 2.5$$

$$\phi_3(q_f) \approx 2.5$$

Treated  
the  
Same!

If  $w_2 = 0$

Then  $\phi_2(q_f)$  may be  $\neq 0$

If  $f$  is blind to age, it possible that  $\phi_{AGE}(q_f) \neq 0!$

Better recourse

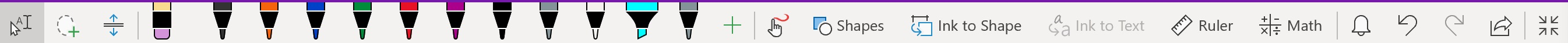
IF

Feature are independent

Better recourse in my opinion

because

Features are dependant



## Conclusion

Going back to the bank example,

Both recourse and R\AI Engineering prefer observational values

But we use interventional almost always.

Also, it's harder to compute.